Shear viscosity from R-charged AdS black holes

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ABSTRACT: We compute the shear viscosity in the supersymmetric Yang-Mills theory dual to the STU background. This is an example of thermal gauge theory with a nonzero chemical potential. The quotient of the shear viscosity over the entropy density exhibits no deviation from the well known result $\eta/s = 1/4\pi$.

KEYWORDS: AdS/CFT correspondence, black holes in string theory.

Contents

1. Introduction

The AdS-CFT correspondence is a calculational scheme that allows to obtain results in strongly coupled gauge theories [1]. The extension to asymptotically AdS spacetimes with a regular horizon is relevant in connection with the thermodynamical propeties of the dual gauge theory at finite temperature. Equilibrium properties match up to numerical factors [2]. Near equilibrium, the low energy behaviour should be governed universally by hydrodynamics. A program to obtain transport coefficients was initiated in [3][4] and a number of results have been obtained since then. The upshot of these calculations was a rather peculiar universal behaviour for the ratio of shear viscosity η and entropy density s of the associated plasma. In all the examples analized, the result

$$\frac{\eta}{s} = \frac{1}{4\pi} \tag{1.1}$$

was found, and in [5] this persistence was related to the universality of the low energy absorption cross section of gravitons [6]. In the context of the AdS-CFT formalism, a proof involving the holographic evaluation of correlators of the energy-momentum tensor was presented in [7]. It extended the above result to supergravity backgrounds for which the relation $R^t{}_t = R^{x_i}{}_{x_i}$ (no sum) among components of the Ricci tensor holds. A significant class of exceptions to this condition include backgrounds which are dual to $\mathcal{N} = 4$ $SU(N)$ supersymmetric Yang-Mills at finite temperature and with a nonzero chemical potential for the $U(1)^3 \subset SO(6)_R$ R-charge. The field strengths of the abelian gauge fields support the difference among components of the Ricci tensor $R^t{}_t - R^{x_i}{}_{x_i} \sim F^I_{rt} F^{I\,rt}$. One such example is the so called STU model, a solution of five dimensional $N = 2$, $U(1)^3$ gauged supergravity first found in [8]. Indeed, in [10] a particular case of this solution was seen to be obtainable from a consistent Kaluza Klein reduction of $D = 10$ type IIB supergravity of a stack of black branes that rotate in the internal S_5, in the near horizon approximation.

The aim of this paper is to computationally fill this small gap. Calculations have been performed by means of the Kubo relation, which yields a direct expression of the shear viscosity in terms of retarded correlators of the energy momentum tensor. For the quotient η/s we find the result (1.1) is shown to persist, a fact which supports the extension to more general backgrounds than those considered in [7].

2. Thermodynamics of the STU background

In this section we shortly review the thermodynamics of the STU model and set up the conventions. Let us start by writing the bare action of five dimensional $N = 2, U(1)^3$ gauged supergravity.

$$I_0 = \frac{1}{2\kappa^2} \int d^5x \sqrt{g} \left(-R - \frac{4}{L^2} \sum_{I=1}^{3} e^{\vec{a}_I \cdot \vec{\phi}} + \frac{1}{2}(\partial\vec{\phi})^2 + \frac{1}{4} \sum_{I=1}^{3} e^{2\vec{a}_I \cdot \vec{\phi}}(F^I)^2 - \frac{\epsilon^{\mu\nu\rho\sigma\lambda}}{4\sqrt{g}} F^1_{\mu\nu} F^2_{\rho\sigma} A^3_\lambda \right) \quad (2.1)$$

where $\vec{\phi} = (\phi_1, \phi_2)$, $\vec{a}_1 = (\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{2}}), \vec{a}_2 = (\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{2}})$ and $\vec{a}_3 = (-\frac{2}{\sqrt{6}}, 0)$. The STU solution depends on two functions of four parameters μ and q_I $(I = 1, 2, 3)$.

$$H_I(r) = \left(1 + \frac{q_I}{r^2}\right) \quad ; \quad \mathcal{H}(r) = \prod_{I=1}^{3} H_I(r) \quad ; \quad f(r) = k - \frac{\mu}{r^2} + \frac{r^2}{L^2}\mathcal{H}(r) \quad (2.2)$$

with which we can write all the field dependences. For example, the metric tensor assumes the form

$$ds_5^2 = -\mathcal{H}(r)^{-2/3} f(r) dt^2 + \mathcal{H}^{1/3}(r) \left(f^{-1}(r) dr^2 + \frac{r^2}{L^2} d\Sigma_{3,k}^2 \right) \quad (2.3)$$

and the scalar and gauge fields exhibit the following profiles

$$\phi_1 = \frac{1}{\sqrt{6}} \log H_1 H_2 H_3^{-2} \quad ; \quad \phi_2 = \frac{1}{\sqrt{2}} \log H_1/H_2 \quad ; \quad A_t^I = \sqrt{\frac{\mu}{q_I} + k} \left(1 - H_I^{-1}\right) \quad (2.4)$$

The discrete parameter $k = 0, \pm 1$ controls the choice of the spatial slices Σ_k of constant curvature

$$d\Sigma_{3,k}^2 \equiv \eta_{ij}^{(k)} dx^i dx^j = \begin{cases} L^2(d\theta_1 + \sin^2\theta_1\, d\theta_2 + \sin^2\theta_1 \sin^2\theta_2\, d\theta_3) & \text{for } k = +1 \\ dx^2 + dy^2 + dz^2 & \text{for } k = 0 \\ L^2(d\theta_1 + \sinh^2\theta_1\, d\theta_2 + \sinh^2\theta_1 \sin^2\theta_2\, d\theta_3) & \text{for } k = -1 \end{cases} \quad (2.5)$$

The case $k = 0$ can be uplifted to the near horizon metric for a stack of plane parallel branes that rotate in the internal S^5 with angular momenta proportional to the charges [9][10]. In the following section we shall investigate this case, but for completeness, in this section we give expressions that encompass the three situations $k = 0, \pm 1$, (to our knowledge these have not appeared elsewhere). We shall denote the volume of the space $d\Sigma_{3,k}^2$ as

$$V_{3,k} = \begin{cases} 2\pi^2 L^3 & \text{for } k = +1 \\ \int d^3x & \text{for } k = 0 \\ 4\pi L^3 \int \sinh^2\theta d\theta & \text{for } k = -1 \end{cases} \quad (2.6)$$

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It will be convenient to trade the nonextremality parameter μ for the horizon radius, $r = r_+$ given as the largest root of $f(r_+) = 0$ or

$$\mu = r_+^2 \left(\frac{r_+^2}{L^2} \mathcal{H}(r_+) + k \right) . \tag{2.7}$$

The entropy density $s = S/V_{3,k}$ is given by the area of the horizon

$$s = \frac{2\pi}{\kappa^2} A = \frac{2\pi}{\kappa^2 L^3} \sqrt{\prod_{I=1}^3 (r_+^2 + q_I)} , \tag{2.8}$$

and for the Hawking temperature one finds

$$T = \frac{1}{2\pi L^2} \frac{2r_+^6 + (kL^2 + \sum_{I=1}^3 q_I)r_+^4 - \prod_{I=1}^3 q_I}{r_+^2 \sqrt{\prod_{I=1}^3 (r_+^2 + q_I)}} . \tag{2.9}$$

There is also a chemical potential conjugate to the physical charge

$$\tilde{q}_I^2 = q_I(r_+^2 + q_I) \left(\frac{1}{L^2 r_+^2} \prod_{J \neq I} (r_+^2 + q_J) + k \right) \tag{2.10}$$

given by the gauge field evaluated at the horizon

$$\Phi^I = \frac{1}{\kappa^2} A_t^I(r) \Big|_{r=r_+} = \frac{1}{\kappa^2} \frac{\tilde{q}_I}{r_+^2 + q_I} \tag{2.11}$$

The thermodynamics of the STU black hole solution has been examined in depth in the past [8][9] where conventional substraction schemes were used in order to extract finite quantities from the asymptotically AdS metric. In [11][12][13] the subject was revised from the point of view of the holographic AdS-CFT renormalization prescription. The holographic renormalization of asymptotically AdS spaces is by now fairly well understood (see [14] and references therein). The addition of a set of covariant boundary counterterms render the action and the correlation functions finite. A nice feature is that these only depend upon the theory under consideration and not the particular solution one is interested in. For pure gravity the set of necessary counterterms has been classified in dimensions up to $d + 1 = 7$ [15]. In the present situation there is a bunch of additional fields present. A systematic construction for an action like the one here was accomplished in [16] (whose conventions we follow) using the Hamilton-Jacobi method of [17] . The result can be written as

$$I = I_0 + I_{GH} + I_{ct} \tag{2.12}$$

where

$$I_{GH} = \frac{1}{\kappa^2} \int_{\Sigma_u} d^4x \sqrt{-h}\, K$$

$$I_{ct} = \frac{1}{\kappa^2} \int_{\Sigma_u} d^4x \sqrt{-h} \left(W(\phi) + \frac{L}{4} R + \mathcal{O}(R^2) \right) \tag{2.13}$$

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with K the trace of the extrinsic curvature and $h_{\mu\nu}$ and R the induced metric and Ricci scalar on the boundary. $W(\phi)$ is the superpotential satisfying

$$V = 2 \sum_i \left(\frac{\partial W}{\partial \phi^i} \right)^2 - \frac{4}{3} W^2. \tag{2.14}$$

This result also appears in [18] a similar setup. In the present case

$$W(\phi) = \frac{1}{L} \sum_{I=1}^{3} e^{-\vec{a}_I \vec{\phi}}. \tag{2.15}$$

We start by listing here the relevant results for the STU background. For the renormalized action

$$I_{ren} = \frac{V_{3,k}}{2\kappa^2 L^2 T} \left(kr_+^2 + \frac{3}{4} k^2 L^2 - \frac{1}{L^2 r_+^2} \prod_{I=1}^{3} (r_+^2 + q_I) \right) \tag{2.16}$$

and for the energy momentum tensor

$$T_{tt} = \frac{1}{2\kappa^2 L} \left(\frac{12}{L^2 r_+^2} \prod_{I=1}^{3} (r_+^2 + q_I) + k \left(12 r_+^2 + 8 \sum_{I=1}^{3} q_I \right) + 3 k^2 L^2 \right) \frac{1}{r^2}$$

$$T_{ij} = \frac{1}{3} T_{tt}\, \eta_{ij}^{(k)}.$$

With this, we can easily obtain the energy density

$$\epsilon = \frac{1}{8\kappa^2 L^3} \left(\frac{12}{L^2 r_+^2} \prod_{I=1}^{3} (r_+^2 + q_I) + k \left(12 r_+^2 + 8 \sum_{I=1}^{3} q_I \right) + 3 k^2 L^2 \right).$$

Making $TI_{ren} = gV_{3,k}$ identifies g with the Gibbs free energy density of the associated grand canonical ensemble, and one can easily check that the expected thermodynamic relations hold

$$g = \epsilon - Ts - \sum_{I=1}^{3} \tilde{q}_I \Phi^I$$

$$dg = -sdT - \sum_{I=1}^{3} \tilde{q}_I d\Phi^I \tag{2.17}$$

3. Shear viscosity from scalar perturbations

In this section we shall set $k = 0$. The above considerations allow us to derive readily some results for transport properties from equilibrium data. We observe that the energy momentum tensor is

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traceless, hence with $P = T_{ii}$ the equality $\epsilon = 3P$ leads to the conformal value for the speed of sound

$$v_s = \left(\frac{\partial P}{\partial \epsilon}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{3}} \tag{3.1}$$

as well as to vanishing bulk viscosity $\zeta = 0$ [19]. Let us turn to a new radial coordinate $u = (r_+/r)^2$ and, after defining $a_I = q_I/r_+^2$, the STU background becomes

$$ds_5^2 = -\mathcal{H}(u)^{-2/3} f(u) dt^2 + \mathcal{H}^{1/3}(u) \left(f^{-1}(u) \frac{r_+^2}{4u^3} du^2 + \frac{r_+^2}{uL^2} d\vec{x}^2 \right) \tag{3.2}$$

$$\mathcal{H}(u) \equiv \prod_{I=1}^{3}(1 + a_I u) \equiv 1 + \alpha_1 u + \alpha_2 u^2 + \alpha_3 u^3$$

$$f(u) = \frac{r_+^2}{uL^2} \left(\mathcal{H}(u) - u^2 \mathcal{H}(1) \right)$$

In field theory there are several strategies to compute the shear viscosity. Probably the most straightforward one is to make use of Kubo's relation

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega i} \left(G_{xy,xy}^A(\omega, 0) - G_{xy,xy}^R(\omega, 0) \right) \tag{3.3}$$

where the retarded Green's function is given by

$$G_{\mu\nu,\lambda\rho}^R(k) = -i \int d^4x e^{-ik \cdot x} \theta(t) \langle [T_{\mu\nu}(x), T_{\lambda\rho}(0)] \rangle \tag{3.4}$$

and $G_{\mu\nu,\lambda\rho}^A(k) = G_{\mu\nu,\lambda\rho}^R(k)^*$. Whereas the original AdS-CFT was designed for Euclidean AdS bulk metrics, the computation of retarded Greens functions only makes sense in Minkowskian AdS. A heuristic prescription to compute the retarded two-point function was put forward in [21]. In order to make use of the Kubo formula, we have to set up a perturbation of the form $h_{xy}(u, x^\mu)$ to the metric and compute the on-shell action as a functional of its boundary value $h_{xy}(0, x^\mu)$. It is easy to check on the equation of motion that the variation of all other supergravity fields can be consistently set to zero. A convenient parametrization is given in terms of $\varphi(u, x^\mu) = g^{xy} h_{xy}$. At linearized order the equation of motion for this polarization is nothing but the equation for a minimally coupled scalar. In a Fourier basis $\varphi(u; x^\mu) = e^{-i\omega t} \varphi_\omega(u)$ we find

$$\varphi_\omega'' + \frac{1 + (1 + \alpha_1 + \alpha_3)u^2 - 2\alpha_3 u^3}{u(u-1)(1 + (1+\alpha_1)u - \alpha_3 u^2)} \varphi_\omega' + \frac{\mathcal{H}(u) \mathfrak{w}^2}{u(u-1)^2(1 + (1+\alpha_1)u - (\alpha_3)^2 u^2)^2} \varphi_\omega = 0. \tag{3.5}$$

where $\mathfrak{w} = \frac{\omega L^2}{2r_+}$. Given the asymptotic normalization $\varphi_\omega(0) = 1$, a regular solution to this equation that satisties the incoming boundary conditions at the horizon $u = 1$ can be found perturbatively in $\mathfrak{w}$

$$\varphi_\omega(u) = (1-u)^{-i\mathfrak{w}\Gamma} \left[1 + \right. \tag{3.6}$$

$$\left. + \frac{i}{2}\mathfrak{w}\Gamma \left(\Delta \log \frac{(\Xi - \alpha_1 - 1 + 2\alpha_3 u)}{(\Xi + \alpha_1 + 1 - 2\alpha_3 u)} \frac{(\Xi + \alpha_1 + 1)}{(\Xi - \alpha_1 - 1)} + \log(1 + (\alpha_1 + 1)u - \alpha_3 u^2) \right) + \mathcal{O}(\mathfrak{w}^2) \right]$$

where the following definitions have been used

$$\Gamma = \frac{\sqrt{1 + \alpha_1 + \alpha_2 + \alpha_3}}{2 + \alpha_1 - \alpha_3} \quad ; \quad \Xi = \sqrt{1 + \alpha_1(2 + \alpha_1) + 4\alpha_3} \quad ; \quad \Delta = -\frac{3 + \alpha_1}{\Xi} \ .$$

Expanding the right hand side of the renormalized action (2.12) up to second order in $\varphi(x, u)$ and expressing the result in terms of the Fourier transform

$$\varphi(x, u) = \int \frac{d^4 k}{(2\pi)^4} e^{ikx} f(k) \varphi_k(u) \tag{3.7}$$

we obtain the following contributions at the regulating surface Σ_u

$$I_0 = \frac{1}{2\kappa^2} \int \frac{d^4 k}{(2\pi)^4} f(k) f(-k) \int_0^1 du (A\varphi_k'' \varphi_{-k} + B\varphi_k' \varphi_{-k}' + C\varphi_k' \varphi_{-k} + D\varphi_k \varphi_{-k}) \tag{3.8}$$

$$I_{GH} = \frac{1}{2\kappa^2} \int \frac{d^4 k}{(2\pi)^4} f(k) f(-k) \left(H\varphi_k \varphi_{-k} + I\varphi_k' \varphi_{-k} \right) \tag{3.9}$$

$$I_{ct} = \frac{1}{2\kappa^2} \int \frac{d^4 k}{(2\pi)^4} f(k) f(-k) \ J\varphi_k \varphi_{-k} \tag{3.10}$$

where the functions A, B, C, D, H, I, and J are given in the appendix. Once the perturbations are set on shell, the bulk action becomes a surface term [20]:

$$I_0 = \frac{1}{2\kappa^2} \int \frac{d^4 k}{(2\pi)^4} f(k) f(-k) \left(\int_0^1 du[E.O.M] \varphi_{-k} + \left. \left(\frac{C - A'}{2} \varphi_k \varphi_{-k} + B\varphi_k' \varphi_{-k} \right) \right|_u^1 \right) \tag{3.11}$$

hence all contributions be arranged in the form of pure boundary terms

$$I_A = \int \frac{d^4 k}{(2\pi)^4} f(k) f(-k) \ \mathcal{F}_A(k, u)|_0^1$$

The prescription for computing retarded Greens function as given in [21] is

$$G_{xy,xy}^R(k) = -2\mathcal{F}(k, u = 0) \tag{3.12}$$

where $\mathcal{F}(k, u) = \sum_A \mathcal{F}_A(k, u)$. Inserting the solution (3.7) into (3.9) (3.10) and (3.11), we obtain

$$\mathcal{F}_0(k, u) = \frac{r_+^4}{2\kappa^2 L^5} \left[\frac{1}{u^2} + \frac{2\alpha_1}{3u} - \frac{3(1 + \alpha_1 + \alpha_3) + 2\alpha_2}{3} - 3i\sqrt{1 + \alpha_1 + \alpha_2 + \alpha_3} \, \mathfrak{w} + \mathcal{O}(\mathfrak{w}^2, u) \right]$$

$$\mathcal{F}_{GH}(k, u) = \frac{r_+^4}{2\kappa^2 L^5} \left[-\frac{4}{u^2} - \frac{8\alpha_1}{3u} + \frac{6(1 + \alpha_1 + \alpha_3) + 2\alpha_2}{3} + 4i\sqrt{1 + \alpha_1 + \alpha_2 + \alpha_3} \, \mathfrak{w} + \mathcal{O}(\mathfrak{w}^2, u) \right]$$

$$\mathcal{F}_{ct}(k, u) = \frac{1}{2\kappa^2} \frac{r_+^4}{L^5} \left(\frac{3}{u^2} + \frac{2\alpha_1}{u} - \frac{3(1 + \alpha_1 + \alpha_3) + \alpha_2}{2} + \mathcal{O}(\mathfrak{w}^2, u) \right)$$

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Adding up we see that the solution is properly renormalized and finite when $u \to 0$ as expected [1]. Moreover we get the retarded Green's function to that order

$$G^R_{xy,xy}(\omega) = \frac{1}{2\kappa^2} \frac{r_+^4}{L^5} \left((1 + \alpha_1 + \alpha_2 + \alpha_3) - 2i\sqrt{1 + \alpha_1 + \alpha_2 + \alpha_3}\, \mathfrak{w} + \mathcal{O}(\mathfrak{w}^2) \right) \quad (3.13)$$

Inserting this expression into the Kubo relation (3.3) gives the the following result for the shear viscosity

$$\eta = \frac{1}{2\kappa^2 L^3} \sqrt{\prod_{I=1}^{3}(r_+^2 + q_I)} \quad (3.14)$$

One may wish to translate this into QFT language by uplifting to IIB supergravity and using the standard dictionary

$$\frac{1}{2\kappa^2} = \frac{V_5}{2\kappa_{10}^2} = \frac{N^2}{8\pi^3 L^3} \quad (3.15)$$

In view of (2.8) we also recover the result (1.1), as promised.

4. Conclusion

We see that the proposed holographic viscosity bound [5] is also saturated in supergravity backgrounds whose dual CFT have a nonvanishing chemical potential.

Just as an aside, in [22] a closed expression for the shear viscosity was proposed relying on the so called "membrane paradigm". Although not rigorously obtained from first principles, this expression is nice both for its simplicity and because it involves properties of the metric close to the horizon. In [23] it was shown that this closed formula reproduced the universal result (1.1) when restricted again to the class of supergravity backgrounds for which $R^t{}_t = R^{x_i}{}_{x_i}$. For the STU geometry, taken plainly, this expression apparently signals a deviation of the above mentioned quotient $\eta/s = 1/4\pi(1 + ...)$. However this formula is not applicable in the present context, as it is based upon the assumption that metric perturbations in the shear channel decouple, whereas in the STU background they do couple to the gauge field. It would be nice to find a modification of that formula that could encompass such mixing.

While this work was in progress we were informed by A. Starinets about a project which overlaps significantly with the one presented here [24]. Also O. Saremi has worked out the shear viscosity in the presence of chemical potential in the context of M-theory backgrounds [25] (see also [26]).

Acknowledgments

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[1] the contribution of the second counterterm in (2.13) starts at $\mathcal{O}(\mathfrak{w}^2)$

A. Coefficients of the renormalized action

The coefficients that enter the renormalized action (3.8) (3.9) and (3.10) are given by the following expressions

$$A = -\frac{r_+^4}{L^5 u} \left[4(1-u)(1+(\alpha_1+1)u - \alpha_3 u^2) \right]$$

$$B = -\frac{r_+^4}{L^5 u} \left[3(1-u)(1+(\alpha_1+1)u - \alpha_3 u^2) \right]$$

$$C = \frac{r_+^4}{L^5} \frac{1}{3u^2 \mathcal{H}(u)} \bigg(-24\alpha_3^2 u^6 + 2\alpha_3(6(1+\alpha_1+\alpha_3) - 11\alpha_2)u^5 + 10((1+\alpha_1+\alpha_3)\alpha_2 - 2\alpha_1\alpha_3)u^4$$

$$+ (8\alpha_1(1+\alpha_1+\alpha_3) + 2\alpha_1\alpha_2 - 6\alpha_3)u^3 + (4\alpha_1^2 + 6(1+\alpha_1+\alpha_3) + 14\alpha_2)u^2 + 22\alpha_1 u + 18 \bigg)$$

$$D = -\frac{r_+^4}{L^5} \left[\frac{\mathcal{H}(u)\mathfrak{w}^2}{u^2(1-u)(1+(\alpha_1+1)u - \alpha_3 u^2)} + \frac{1}{3u^3 \mathcal{H}(u)^2} \bigg((\alpha_3\alpha_2(1+\alpha_1+\alpha_2+\alpha_3) - 2\alpha_3^2\alpha_1)u^7 \right.$$

$$+ 2\alpha_3(2\alpha_1(1+\alpha_1+\alpha_3) - 3\alpha_3)u^6 + ((9\alpha_3 + \alpha_1\alpha_2)(1+\alpha_1+\alpha_3) - 4\alpha_3\alpha_1^2 - \alpha_1\alpha_2^2 - 3\alpha_3\alpha_2)u^5$$

$$+ (4\alpha_2(1+\alpha_1+\alpha_3) - 16\alpha_3\alpha_1 - 2\alpha_2^2 - 4\alpha_1^2\alpha_2)u^4 + ((1+\alpha_1+\alpha_3)\alpha_1 - 2\alpha_1^3 - 15\alpha_1\alpha_2 - 12\alpha_3)u^3$$

$$\left. - (10\alpha_1^2 + 12\alpha_2)u^2 - 14\alpha_1 u - 6 \bigg) \right]$$

$$H = \frac{r_+^4}{L^5} \frac{1}{3u^2 \mathcal{H}(u)} \bigg((3(1+\alpha_1+\alpha_3)\alpha_3 - \alpha_2\alpha_3)u^5 + (4(1+\alpha_1+\alpha_3)\alpha_2 - 8\alpha_3\alpha_1)u^4$$

$$+ (5(1+\alpha_1+\alpha_3)\alpha_1 - 7\alpha_1\alpha_2 - 12\alpha_3)u^3 + (6(1+\alpha_1+\alpha_3) - 8\alpha_1^2 - 10\alpha_2)u^2 - 20\alpha_1 u - 12 \bigg)$$

$$I = \frac{r_+^4}{L^5 u} \left[4(1-u)(1+(1+\alpha_1)u - \alpha_3 u^2) \right]$$

$$J = \frac{r_+^4}{L^5} \frac{1}{u^2 \mathcal{H}(u)^{1/2}} \sqrt{(1-u)(1+(1+\alpha_1)u - \alpha_3 u^2)} \; (3 + 2\alpha_1 u + \alpha_2 u^2) \qquad \text{(A.1)}$$

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